Application of fractional chaotic system to explore human gait dynamics

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Overview

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- **Fractional Rössler Oscillator:** The Rössler oscillator, renowned for its complex dynamics, is extended using fractional derivatives, adding insight into chaotic behavior within the context of human gait analysis.
- **Applications in Exoskeletons:** The Particularly-Shaped Adaptive Oscillator (PSAO) model, based on fractional chaos, synchronizes with users' gait in exoskeletons, reducing metabolic walking costs and enhancing efficiency in assisted walking [15].

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- **Grünwald-Letnikov Characterization:**

$$D_{0,t}^{\alpha} f(t_i) = \lim_{\Delta t \to 0} \frac{1}{\Delta t^{\alpha}} \sum_{n=0}^{\infty} (-1)^n \binom{\alpha}{n} f(t_i - n\Delta t) \tag{1}$$

where $\binom{\alpha}{n}$ is the generalized binomial coefficient [9].

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  where $\binom{\alpha}{n}$ is the generalized binomial coefficient [9].
- **Computational Advantages:** Grünwald–Letnikov method extends the Euler method and incorporates fractional binomial coefficients [10]. The method offers stability and computational efficiency, making it suitable for numerical simulations and computations.

Fractional Rössler Oscillator

- **Equations:** Here $0 < \alpha < 1$ is a real number,and a,b,c are the parameters where a and α are bifurcation parameters. D_0^α denotes the fractional derivative following the GL definition [11].

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\begin{aligned}
D_0^\alpha x &= -y - z \\
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- Studying these fractional dynamics offers deeper insights into nonlinear systems, chaos theory, and has implications for understanding complex phenomena like human gait.

Fixed Point Analysis and Stability

- Two fixed points $[S_{p+}(x_{Fp^+}, y_{Fp^+}, z_{Fp^+})$ and $S_{p-}(x_{Fp^-}, y_{Fp^-}, z_{Fp^-})]$:

$$\left(\frac{c \pm \sqrt{c^2 - 4ab}}{2}, -\frac{-c \pm \sqrt{c^2 - 4ab}}{2a}, \frac{c \pm \sqrt{c^2 - 4ab}}{2a}\right).$$

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- **Jacobian:**

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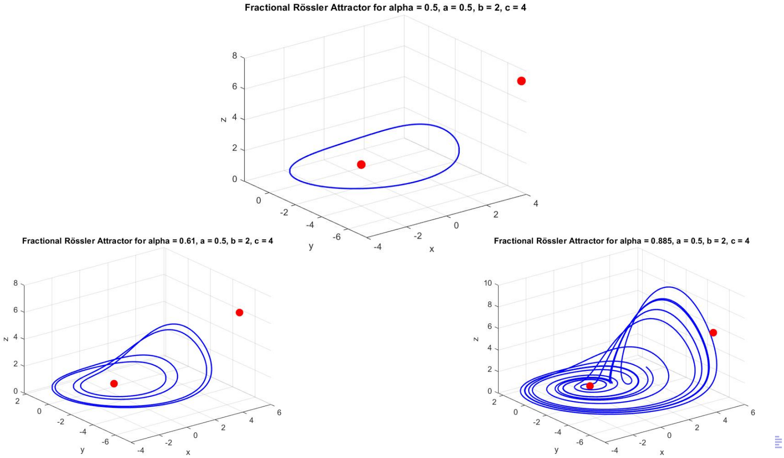
- **Characteristic equation:**

$$\lambda^3 - \lambda^2(a + x - c) + \lambda(ax + 1 + z - ac) + c - x - az = 0 \qquad (3)$$

(Where λ is the eigenvalue, a, b, c are parameters).

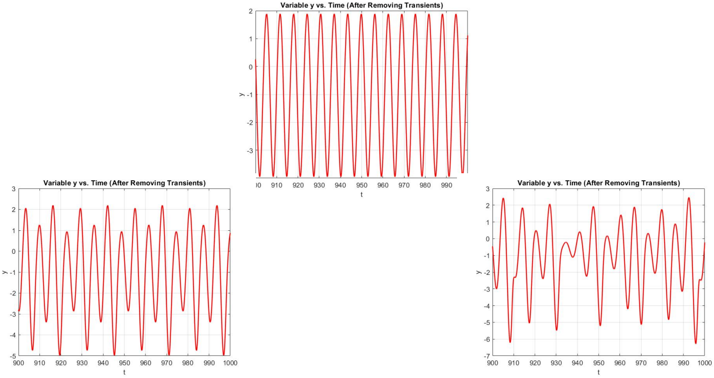
Plot of Oscillator and fixed points

Fractional Rössler attractor when fractional order (α) is increased from 0.5 to 0.885 by keeping other parameters constant [Fixed points are shown with red dots]:



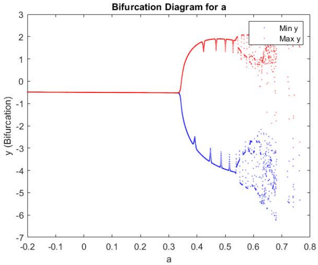
Plot of y(t) vs. t

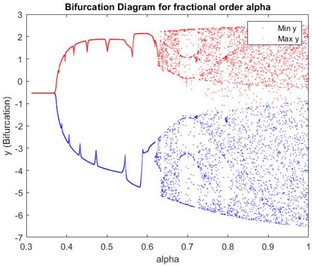
Figures show the plots of y(t) vs t for $\alpha = 0.5$ (periodic), $\alpha = 0.61$(periodic), $\alpha = 0.885$ (chaotic) (LHS TO RHS (anti-clockwise)):



Bifurcation Diagram

Bifurcation diagram showing the minimum and maximum values of y when $(-0.2 < a < 1)$ [LHS] and $(0.3 < \alpha < 1)$ [RHS] when other parameters are constant. Here last 500 points of y values are considered.





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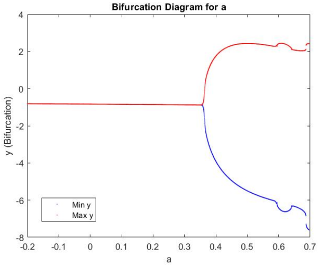

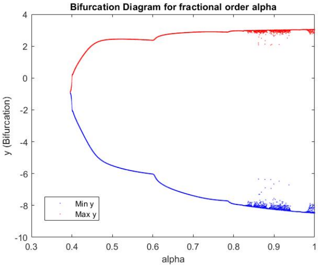

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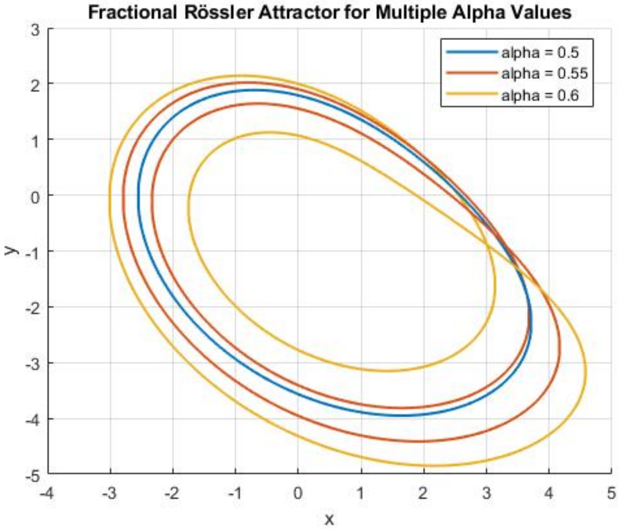
- We have demonstrated the effect the fractional parameter for the emergence of Hopf Bifurcation which gives rise to a limit cycle and eventually leads to chaos.
- The evolution of limit cycle gives us an insight for the human gait analysis which exhibits also an oscillatory behaviour.

Limit Cycles

Limit Cycles when the fractional parameter varies from 0.5 to 0.6



Matignon Criterion and Eigenvalues

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- For $\alpha = 0.61$, one of the eigenvalues corresponding to the fixed point $(0.2679, -0.5359, 0.5359)$ is $0.1802 + 0.9653i$. Hence, $|arg(0.1802 + 0.9653i)| = 5.9308$ and $\frac{\alpha\pi}{2} = 0.9581$.

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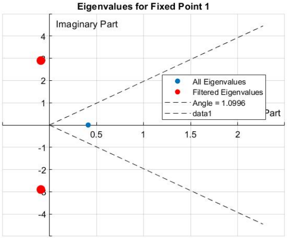
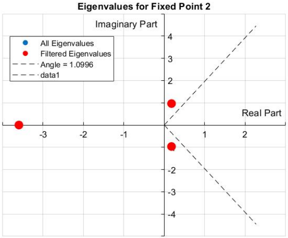
- Similarly, another eigenvalue is $(0.1802 - 0.9653i)$. Hence, $|arg(0.1802 - 0.9653i)| = 5.9308$. The last eigenvalue is $(-3.5924 + 0.0000i)$. Hence, $|arg(-3.5924 + 0.0000i)| = 0$.

Eigenvalues

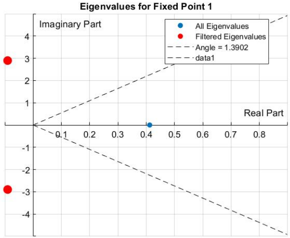
Eigenvalues corresponding to S_p^+ and S_p^- for $\alpha = 0.7$ (before hopf bifurcation) $[a = 0.5, b = 2, c = 4]$:

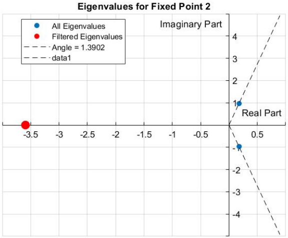




Eigenvalues

Eigenvalues corresponding to S_p^+ and S_p^- for $\alpha = 0.885$ (after hopf bifurcation) $[a = 0.5, b = 2, c = 4]$:





Lyapunov Exponents

Formula used [18]:

$$\text{Lyapunov Exponent} = \frac{1}{N \cdot \Delta t} \sum_{i=2}^{N} \ln \left(\frac{\|\mathbf{P}_i\|}{\|\mathbf{P}_1\|} \right) \tag{4}$$

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- In this study, for alpha = 0.885, a = 0.5, b = 2, c = 4, the positive Lyapunov exponent is equal to 0.57211.

Application to Human Gait System

- **Complexity of Human Gait:** Human gait, as an oscillatory system, shares fundamental characteristics with the dynamic behavior of the Rössler oscillator. However, the unpredictable non-linearity and uncertain environment make practical motion capture challenging.

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- **Application of Fractional Rössler Oscillator:** The utilization of fractional Rössler oscillators in modeling human gait is unconventional but promising. These chaotic systems can effectively capture the complexity and nonlinear dynamics of gait, aiding in the analysis of both symmetric and asymmetric gaits and potential gait disorder treatments. Another control parameter α helps to tweak the values governing the system dynamics more and provides better flexibility to understand human gaits.

Application to Human Gait System

- **Multifaceted Benefits:** Fractional calculus allows the incorporation of memory effects, essential for accurately representing gait's adaptive nature. Chaotic systems align with the stochastic variability inherent in gait, and fractional-order systems hold promise for nonlinear control in gait rehabilitation and assistive devices, offering individualized solutions for patients (with orthopedic diseases, limping). Additionally, it aims to unravel the potential of fractional Rössler oscillators in enhancing our understanding and improvement of natural gait patterns.

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- **Increased Adaptability:** Fractional-order systems offer potential advancements in nonlinear control, enabling more adaptable control in the domain of gait analysis.

Conclusion and Future Scope

- **Fractional Derivative Parameter Variation:** Adjusting the fractional derivative parameter in the fractional Rössler oscillator results in qualitatively different dynamical behaviors, following a Hopf-like bifurcation scheme.

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- **Relevance to Human Gait:** These findings are significant in the context of human gait design and improvement, as they suggest that perturbations in gait dynamics can be controlled and modeled using fractional-order chaotic oscillators.

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Thank you!