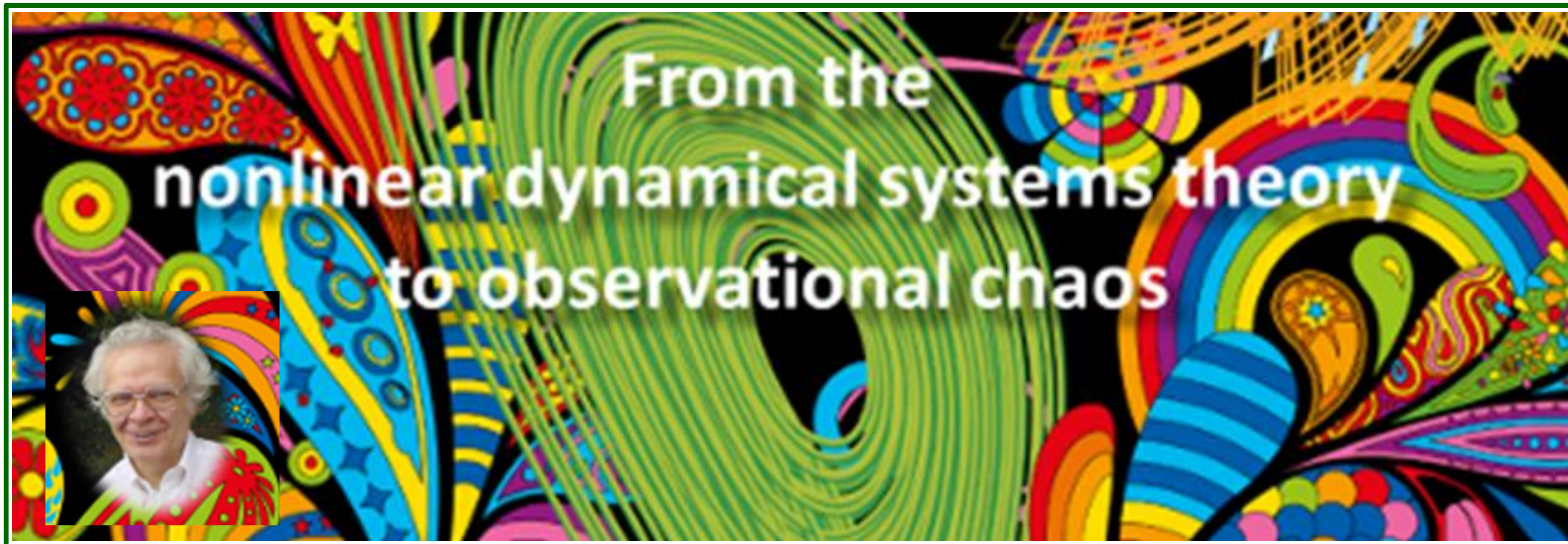

Would chaotic dynamical systems be more beautiful if they were useless?

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The dawn of chaotic dynamical systems

(from early beginning to nowadays)

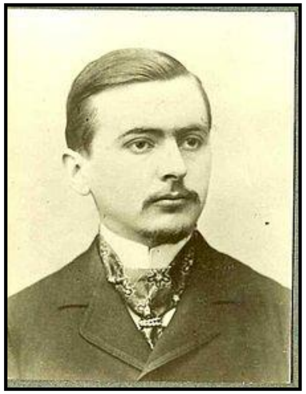
The History of chaotic iterations (discrete dynamical systems) and chaotic differential equations (continuous dynamical systems) is strongly intertwined

The dawn of chaotic iterations (I)

Henri Poincaré

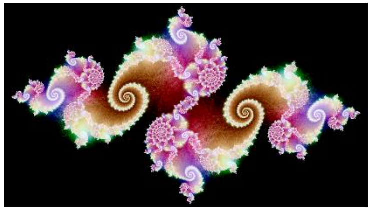
(1854-1912)

The study of nonlinear dynamics is relatively recent with respect to the long historical development of the early mathematics since the Egyptian and the Greek civilizations. The beginning of this study can be traced to the phenomenal work of Henri Poincaré. The Poincaré map being an essential tool linking differential equations and mappings.



Pierre Fatou
1878-1929

Concerning iterations theory, one has to include in this field of research the pioneer works of Gaston Julia and Pierre Fatou related to one-dimensional maps with a complex variable, near a century ago.



"Julia set"

Gaston Julia
1893-1978

The dawn of chaotic iterations (II)

Igor Gumowsky

Christian Mira

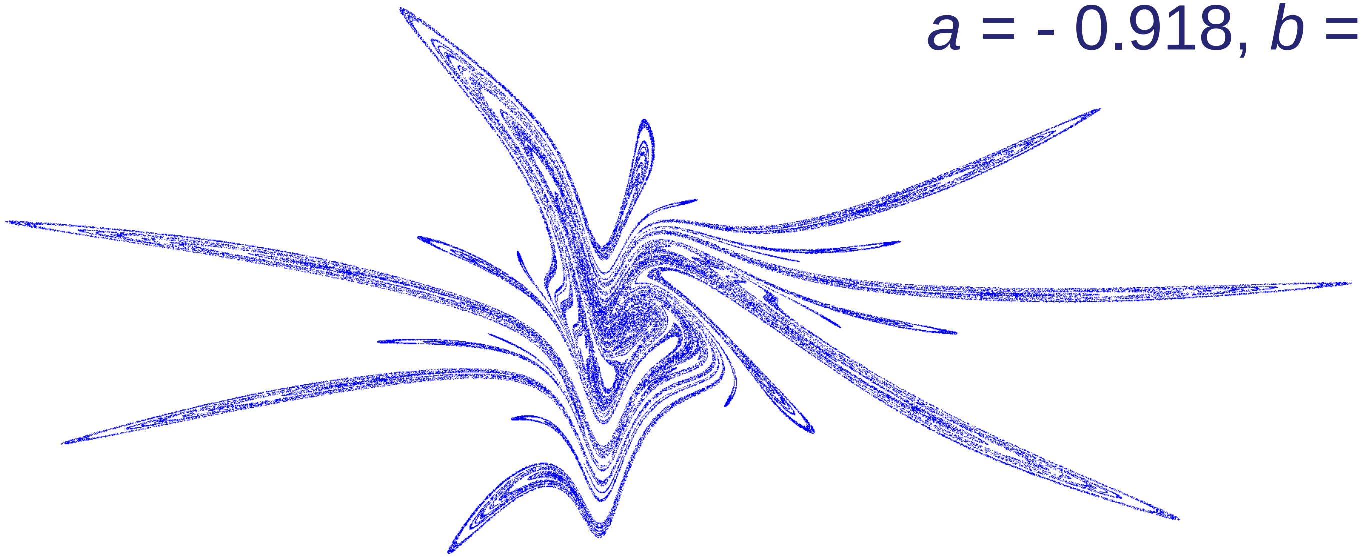
In France Igor Gumosky and Christian Mira began their mathematical research in 1958. They produced a considerable work on the matter (theory of boxes in the boxes for example). Among their discoveries one can emphasize on their family of attractors from an aesthetic point of view (of course it is only a microscopic point of view of what they have produced)

The Gumowski-Mira attractor:

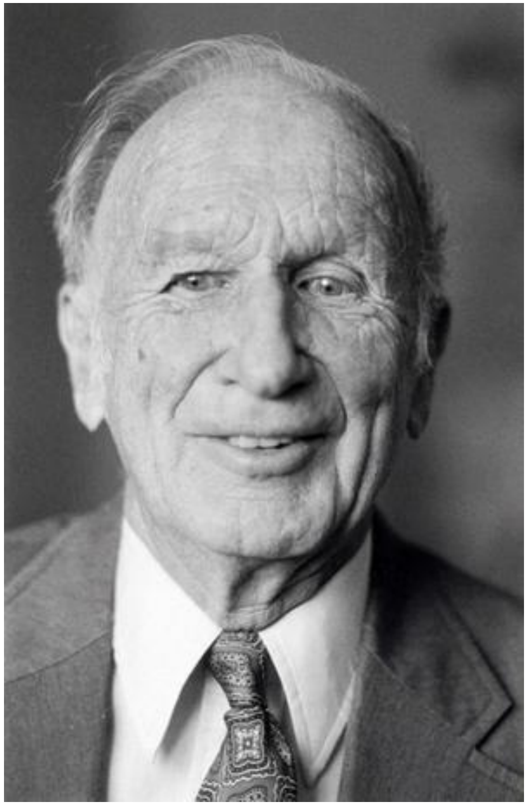
$$\begin{cases} x_{n+1} = f(x_n) + by_n, \\ y_{n+1} = f(x_{n+1}) - x_n, \end{cases} \quad with \quad f(x) = ax + 2(1-a)\frac{x^2}{1+x^2},$$

is sensitive to slight changes of parameters *a* and *b*

The dawn of chaotic iterations (III)

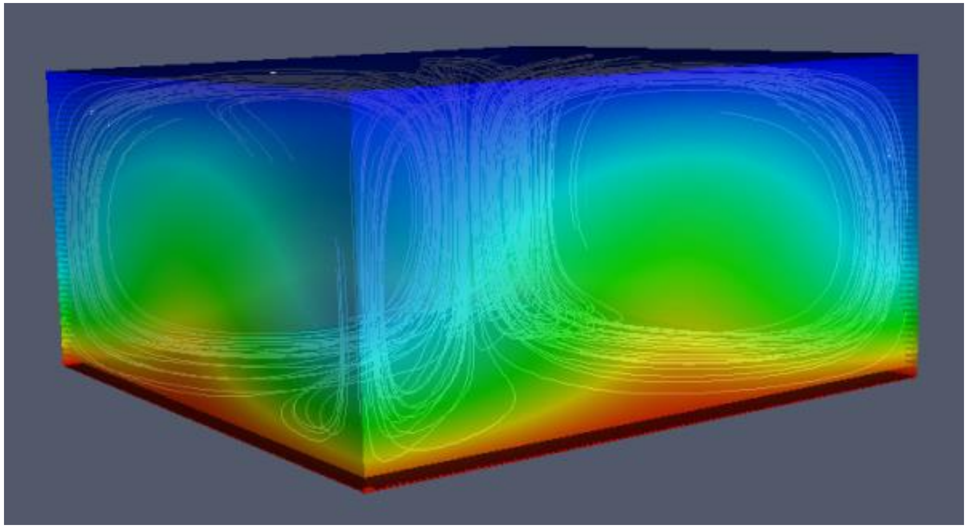


The dawn of chaotic differential equations (I)



Edward Lorenz (1917-2008)

Apart of mathematical research, first came the work of Edward Lorenz a meteorologist who studied the Rayleigh-Bénard problem in 1963

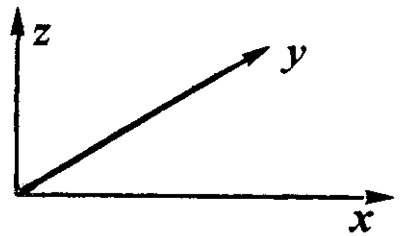


Motion of a flow heated from below

The dawn of chaotic differential equations (II)

Flow equations in a physical coordinate system (constant along *y*)

$$\begin{cases} \dfrac{\partial(\Delta\psi)}{\partial t} = \dfrac{\partial(\psi,\Delta\psi)}{\partial(x,z)} + \sigma\Delta^2\psi + \dfrac{\partial\theta}{\partial x} \\ \dfrac{\partial\theta}{\partial t} = -\dfrac{\partial(\psi,\theta)}{\partial(x,z)} + \rho\dfrac{\partial\psi}{\partial x} + \Delta\theta \end{cases}$$



ρ Rayleigh number, σ Prandtl number, ψ ***(x,t,z)*** stream function, θ ***(x,t,z)*** temperature perturbation vs linear profile

Discretization of equations In Fourier series:

$$\Psi(x,z,t) = \sum_{\substack{m,n\\ m\neq 0}} \Psi_{m,n}(t)\sin(amx)\sin(nz)$$

$$\theta(x,z,t) = \sum_{\substack{m,n\\ m\neq 0}} \theta_{m,n}(t)\cos(amx)\sin(nz)$$

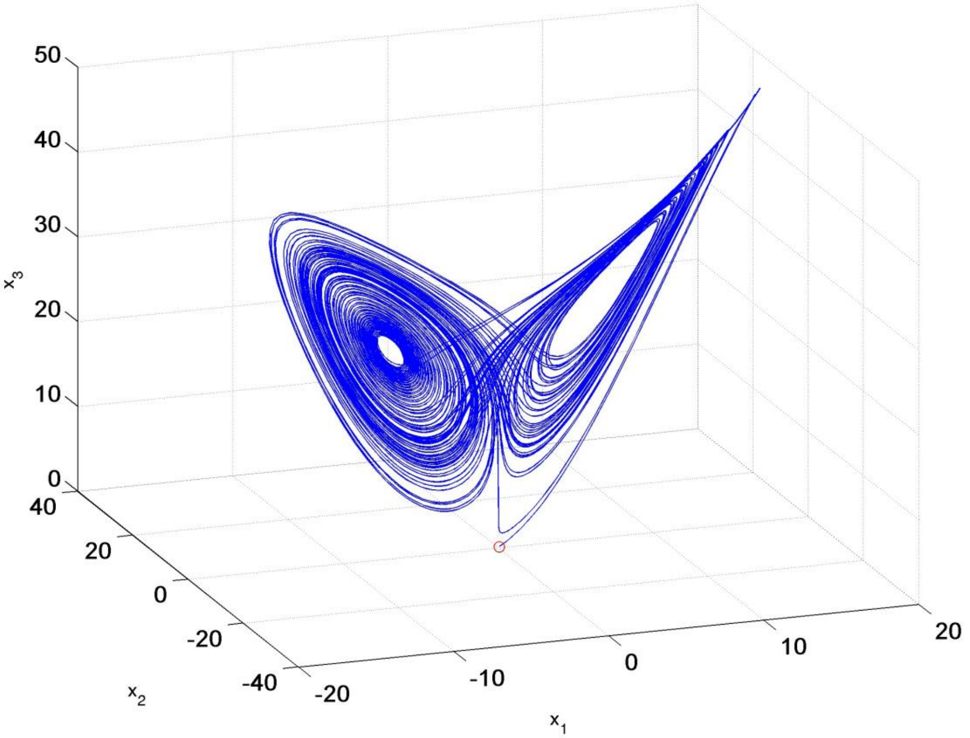
The dawn of chaotic differential equations (III)

Lorenz Attractor (1963)

Flow equations in a physical coordinate system (constant along *y*)

$$\begin{cases} \dot{x}_1 = -\sigma x_1 + \sigma x_2 \\ \dot{x}_2 = \rho x_1 - x_2 - x_1 x_3 \\ \dot{x}_3 = x_1 x_2 - \beta x_3 \end{cases}$$

$$\sigma = 10 \; , \rho = 28 \; , \beta = \frac{8}{3}$$



"Butterfly effect"

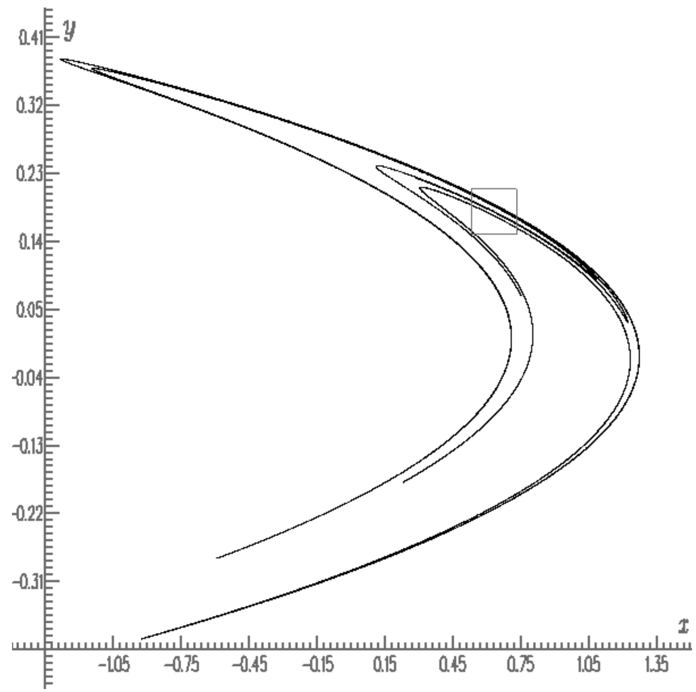
The dawn of chaotic iterations (IV)

Two-Dimensional discrete dynamical systems: Hénon mapping (1976)

Metaphoric model of Poincaré map of Lorenz equation.

$$\mathcal{H}_{a,b} : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$$

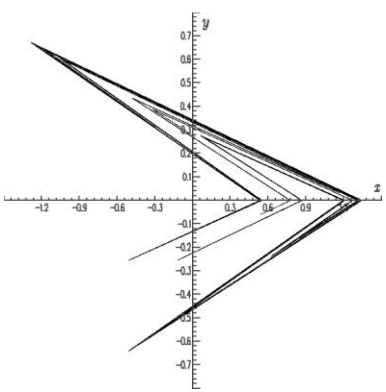
$$\mathcal{H}_{a,b} \left(\begin{array}{c} x \\ y \end{array} \right) = \left(\begin{array}{l} 1 - ax^2 + y, \\ bx. \end{array} \right) \quad a = 1.4, \quad b = 0.3$$



Associated difference equation

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = \begin{pmatrix} y_n + 1 - a x_n^2 \\ b x_n \end{pmatrix}$$ with initial value: $$\begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$

Linearized version in 1978



$$L_{a,b} : \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y + 1 - a|x| \\ bx \end{pmatrix}$$

$$a = 1.7, \quad b = 0.5$$

The dawn of chaotic differential equations (IV)

Rössler's chimical multivibrator (1976)

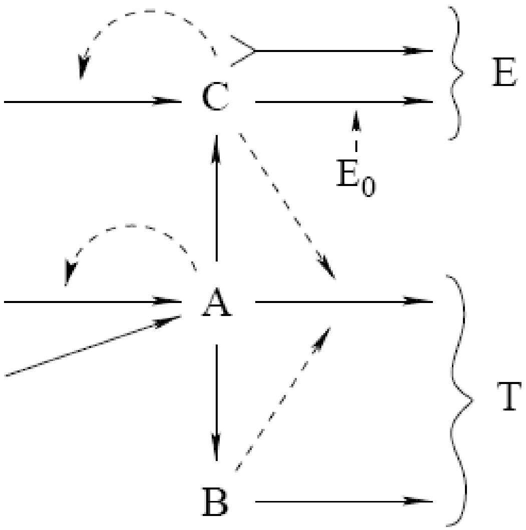


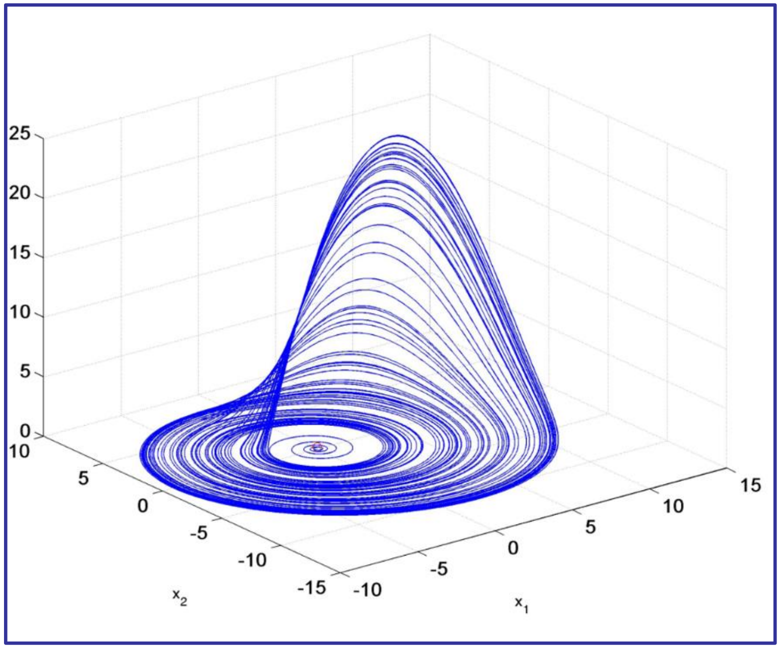
Fig. 12. Combination of an Edelstein switch with a Turing oscillator in a reaction system producing chaos. E = switching subsystem, T = oscillating subsystem; constant pools (sources and sinks) have been omitted from the scheme as usual. (Adapted from [Rössler, 1976a].)

(from Christophe Letellier and Valérie Messager (2010))

The dawn of chaotic differential equations (V)

The Rössler attractor (1976)

In 1976, O. E. Rössler followed a different direction of research to obtain a chaotic model. Considering that, due to extreme simplification used by Lorenz in order to obtain his equation, there is no actual link between this equation and the Rayleigh-Benard problem from which it originated. He followed a new way in the study of a chemical multi-vibrator.



$$
\begin{cases}
\dot{x}_1 = -x_2 - x_3\,, \\
\dot{x}_2 = x_1 + a x_2\,, \\
\dot{x}_3 = b + x_3(\,x_1 - c\,),
\end{cases}
$$

$$a = 0.2\ , b = 0.2\ , c = 5.7$$

Conference for the 80th birthday of
Otto Rössler, October 9-11, 2023

The dawn of chaotic iterations (V)

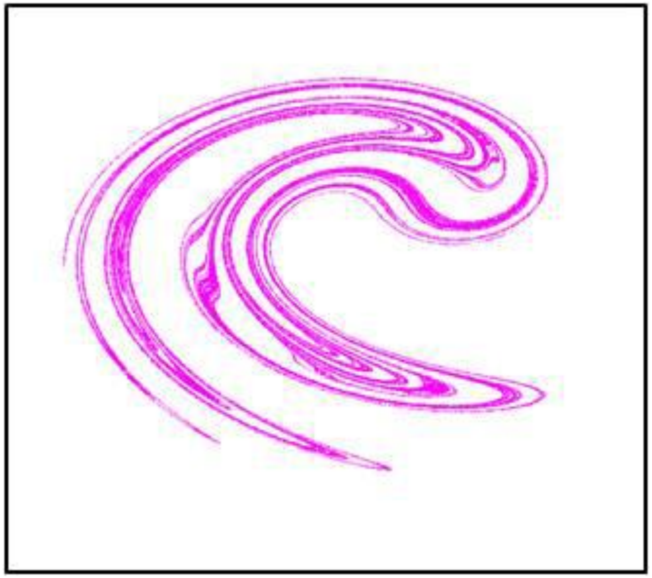
In Japan the Hayashi's School (with disciples like Ikeda, Ueda and Kawakami) in the same period, were motivated by applications to electric and electronic circuits. Mappings were used as models of behavior of electric circuits.

The Ikeda attractor (1980):
has a chaotic attractor when *u* ≥ 0.6

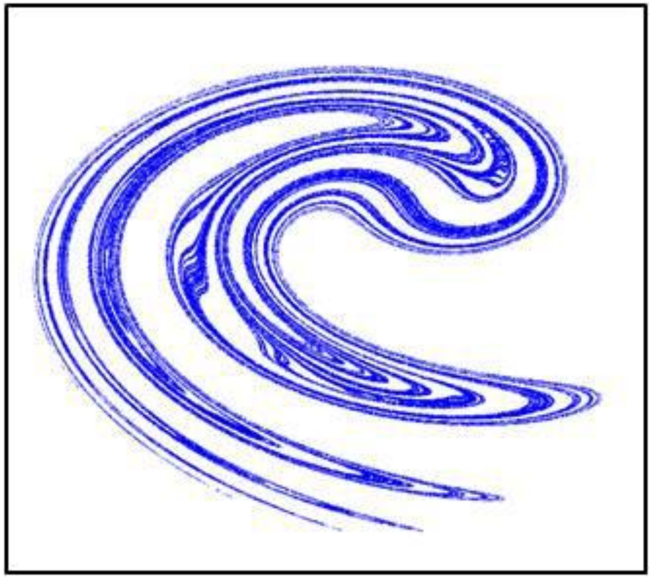
$$\begin{cases} x_{n+1} = 1 + u(x_n \cos t_n - y_n \sin t_n) \\ y_{n+1} = u(x_n \sin t_n + y_n \cos t_n) \end{cases},$$

$$\text{with} \quad t_n = 0.4 - \frac{6}{1 + x_n^2 + y_n^2}$$

u = 8.6



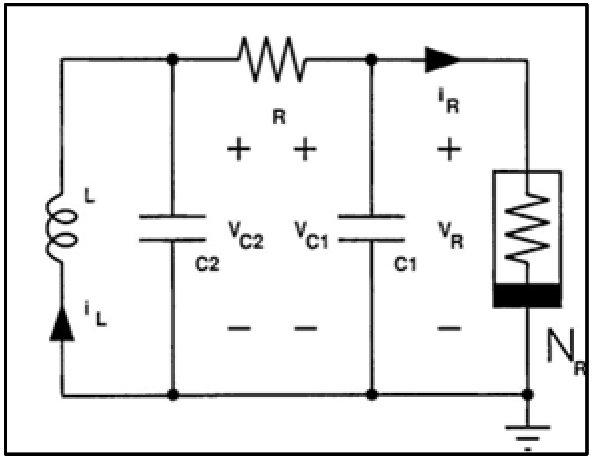
u = 8,9



The dawn of chaotic differential equations (VI)

The Chua attractor (1983)

In 1983, L. O. Chua, invented a very simple electric circuit producing chaos



$$\begin{cases} \dot{x} = \alpha(y - \Phi(x)) \\ \dot{y} = x - y + z \\ \dot{z} = -\beta y \end{cases}$$

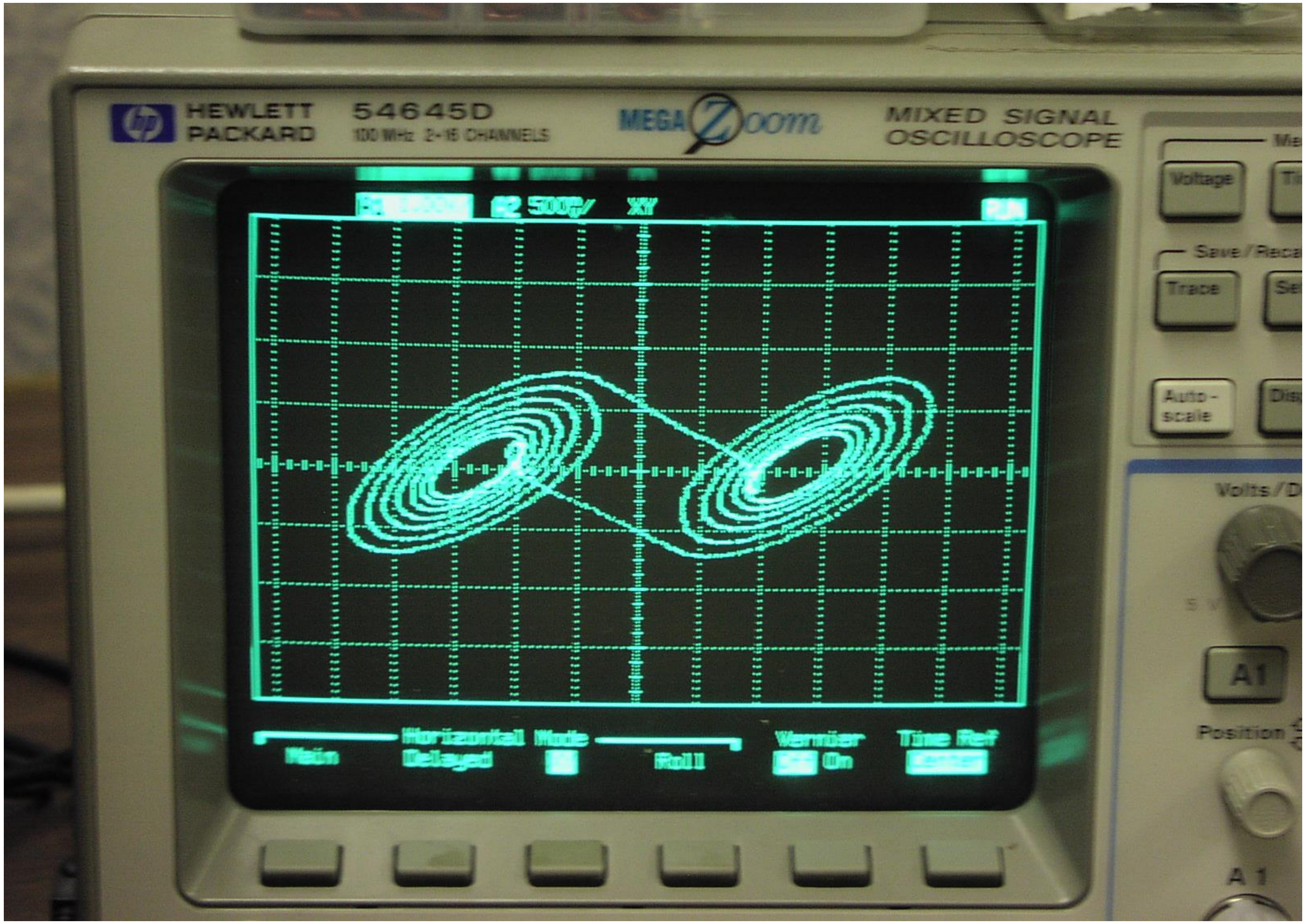
$$\phi(x) = x + g(x) = m_1 x + \frac{1}{2}(m_0 - m_1)[|x+1| - |x-1|]$$

$$\alpha = 15.60, \beta = 28.58, m_0 = -\frac{1}{7}, m_1 = \frac{2}{7}$$

The dawn of chaotic differential equations (VII)

Chua attractor on oscilloscope

Contrarily to Lorenz and Rössler attractor, Chua circuit corresponds to a real device



The dawn of chaotic differential equations (VIII)

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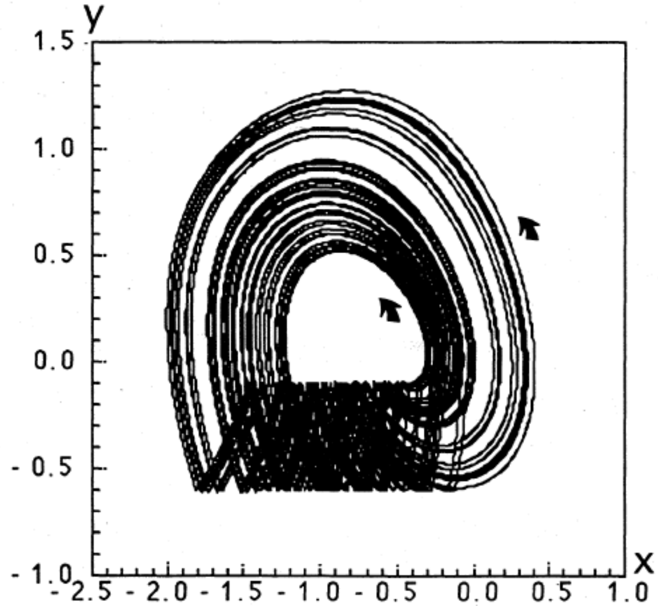
Fig. 1. Alpazur oscillator.



Spiral-linear attractor published in 1992

【SL 系】半平面 $H = \{(x, y) \in R^2 \mid y \geq -1\}$ 上では

$$\begin{cases} \dfrac{dx}{dt} = \sigma x - y \\ \dfrac{dy}{dt} = x + \sigma y \end{cases} \tag{7}$$

また半平面 $B = \{(x, y) \in R^2 \mid y \leq -1 + \alpha, 0 \leq \alpha \leq 1\}$ 上では

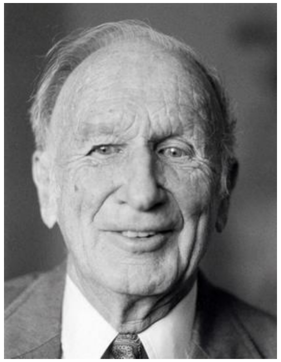
$$\begin{cases} \dfrac{dx}{dt} = 0 \\ \dfrac{dy}{dt} = 1 \end{cases} \tag{8}$$

で定義される合成系，Spiral-Linear 系 (SL 系)の運動と，この系の 2 パラメータ (α, σ) 分岐問題を考えよう．

The dawn of chaotic dynamical systems

In the last 50 years long history of chaotic iterations leading to the new concept of strange attractors, and corresponding chaotic differential systems, one can mention few important dates:

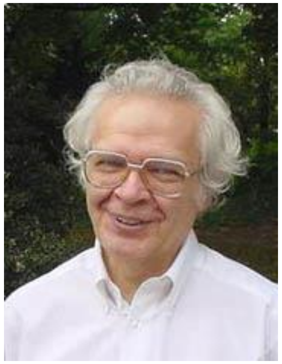
Sharkovsky order
1962



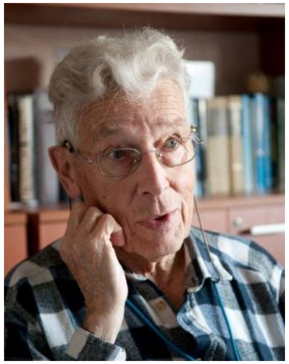
Lorenz attractor
1963

Ruelle “strange attractor” 1970

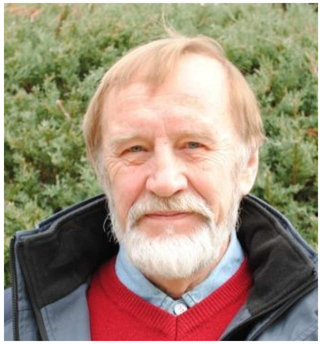
Yorke “Chaos”
1975



Rössler attractor
1976



Hénon map
1976



Belykh map
1976

Chua attractor
1983

From theory to applications

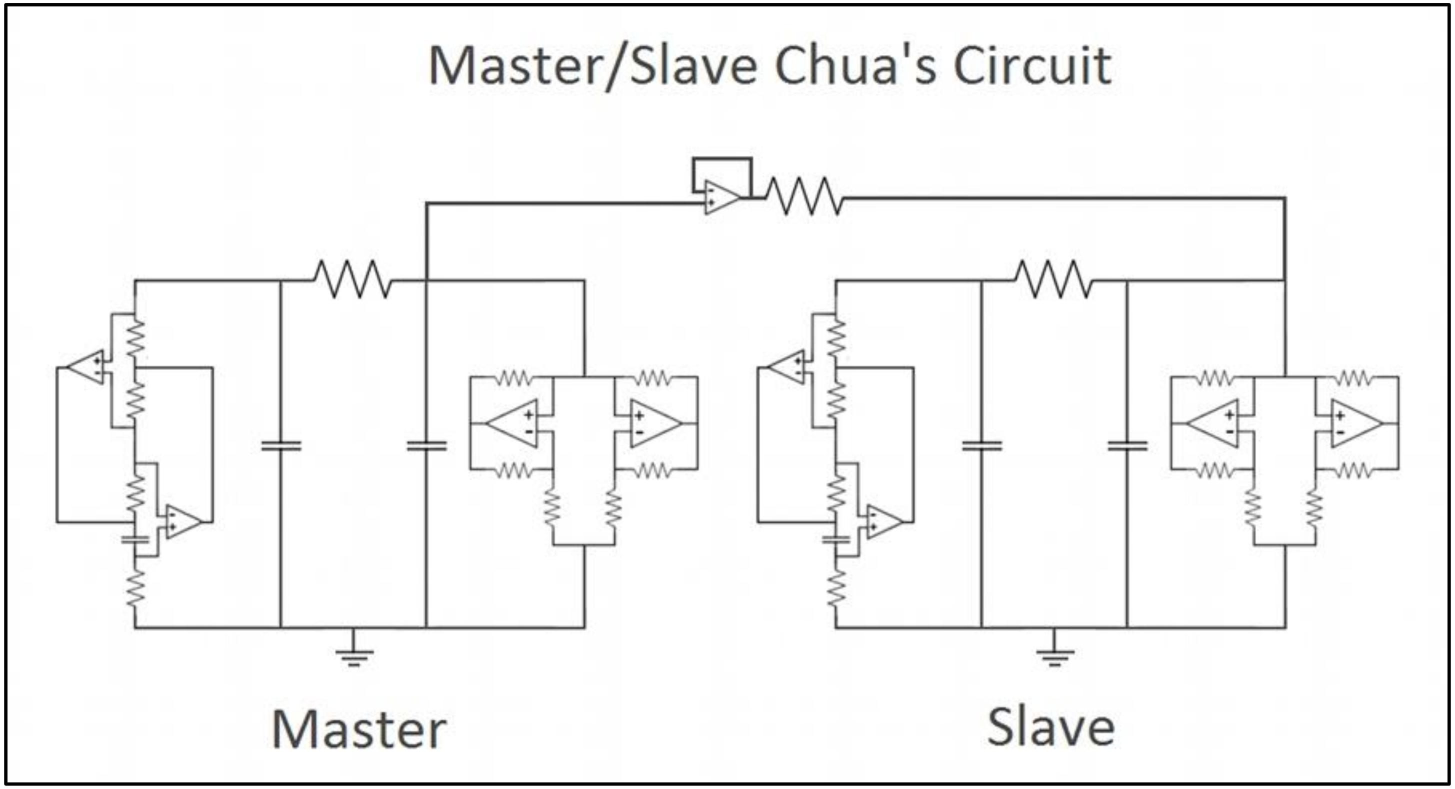
Since the last 50 years many many papers were published in pure mathematics or statistical physics concerning research on properties of nonlinear maps and chaotic iterations (entropy, ergodicity, Lyapunov and Hurst exponents, invariant measure, fractal dimensions, border-collision, …)

However applications of nonlinear mappings in applied mathematics and engineering, biology, physics, …began only 20 years after.

- Secure communications,
- Chaos to randomness : Chaotic Pseudo Random Generators,
- Cryptography based Chaos,
- Global optimization (Particle swarm optimization (PSO)),
- Evolutionary Algorithms,
- Memristors
- Economy

Secure communication via chaotic synchronization

The first example of the use of chaos for cryptographic purpose goes back to the early 90' when L. Pecora and T. Carroll found how to synchronize chaotic systems. This discovery was an unexpected breakthrough for applications.
A first reported experimental secure communication system via chaotic synchronization using Chua's circuit was built two years after (1992).



Secure communication via chaotic synchronization

The signal recovered from this system which uses the Chua circuit, contained some inevitable noise which degrades the fidelity of the original message.

The system was soon improved (1993), by cascading the output of the receiver in the original system, into an identical copy of this receiver:

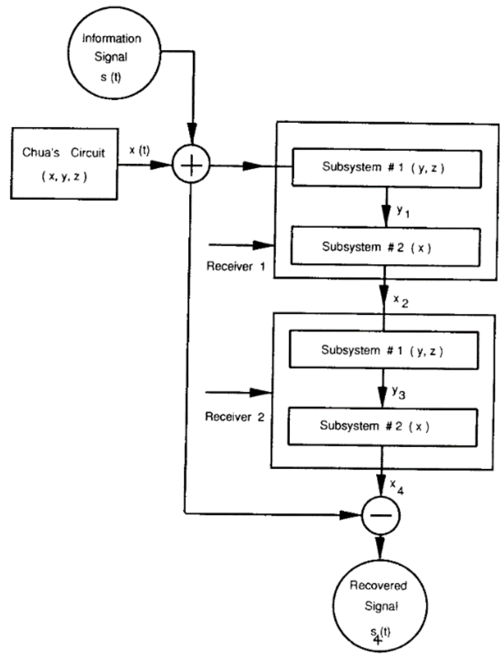


Fig. 2. Block diagram of the electronic circuit implemented in Fig. 1.

International Journal of Bifurcation and Chaos, Vol. 3, No. 5 (1993) 1319–1325

SECURE COMMUNICATIONS VIA CHAOTIC SYNCHRONIZATION II: NOISE REDUCTION BY CASCADING TWO IDENTICAL RECEIVERS

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Received July 15, 1993

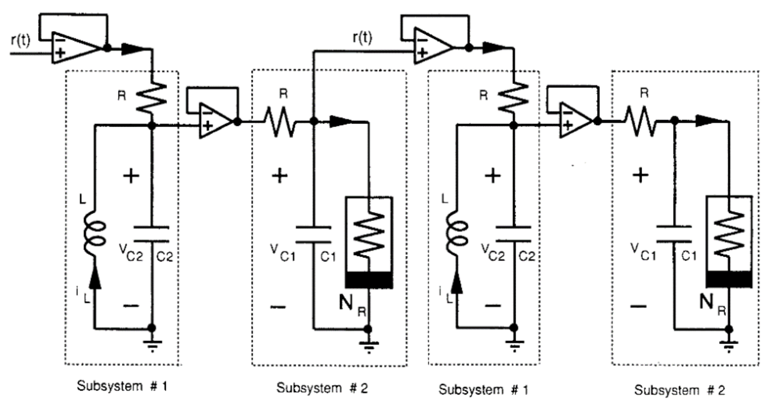

Fig. 1. Electronic circuit implementation of the two-stage "receiver" consisting of two identical copies of the circuit given in Fig. 2(b) of Kocarev *et al.* [1992].

From chaotic attractors
To Pseudo Random Number Generators (PRGN)

The route from chaos to pseudo-randomness via chaotic or mixing undersampling

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EMERGENCE OF RANDOMNESS FROM CHAOS

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Received December 15, 2011

Chaotic and/or mixing undersampling

Step 1: Ultra-weak coupling of 1-D maps

$f(x) = 1 - 2|x|$ $\quad x_{n+1} = 1 - 2|x_n|$ example with the symmetric tent map

$$\begin{cases} x^1_{n+1} = (1-3\varepsilon_1)f(x^1_n) + \varepsilon_1 f(x^2_n) + \varepsilon_1 f(x^3_n) + \varepsilon_1 f(x^4_n) \\ x^2_{n+1} = \varepsilon_2 f(x^1_n) + (1-3\varepsilon_2)f(x^2_n) + \varepsilon_2 f(x^3_n) + \varepsilon_2 f(x^4_n) \\ x^3_{n+1} = \varepsilon_3 f(x^1_n) + \varepsilon_3 f(x^2_n) + (1-3\varepsilon_3)f(x^3_n) + \varepsilon_3 f(x^4_n) \\ x^4_{n+1} = \varepsilon_4 f(x^1_n) + \varepsilon_4 f(x^2_n) + \varepsilon_4 f(x^3_n) + (1-3\varepsilon_4)f(x^4_n) \end{cases}$$

Ultra-weak coupling means

$\varepsilon_i \approx 10^{-7}$ for floating points or $\varepsilon_i \approx 10^{-14}$ for double precision numbers

Ultra-weak coupling is efficient in order to restore numerically the chaotic properties of chaotic mappings, avoiding any numerical collapse

Chaotic and/or mixing undersampling

Step 2: Chaotic and mixing under sampling

Example in 4-D: Let be three thresholds $-1 < T_1 < T_2 < T_3 < 1$
instead of using directly the coupled sequences

$$\left(x_0^1,\ x_1^1,\ x_2^1,\ \ldots,\ x_n^1,\ x_{n+1}^1,\ \ldots\right) \quad \left(x_0^2, x_1^2, x_2^2, \ldots, x_n^2, x_{n+1}^2, \ldots\right) \quad and$$

$$\left(x_0^3,\ x_1^3,\ x_2^3,\ \ldots,\ x_n^3,\ x_{n+1}^3,\ \ldots\right)$$

One mixes and samples those sequences using the fourth one:

$$\left(x_0^4, x_1^4, x_2^4, \ldots, x_n^4, x_{n+1}^4, \ldots\right)$$

using: $$\overline{x_q} = \begin{cases} x_n^1 & iff \quad x_n^4 \in \left]\, T_1, T_2 \right[\\ x_n^2 & iff \quad x_n^4 \in \left[\, T_2, T_3 \right[\\ x_n^3 & iff \quad x_n^4 \in \left[\, T_3, 1 \right[\end{cases}$$

In order to obtain: $\left(\overline{x_0}, \overline{x_1}, \overline{x_2}, \cdots, \overline{x_q}, \overline{x_{q+1}}, \cdots\right)$ which are pseudo-random.

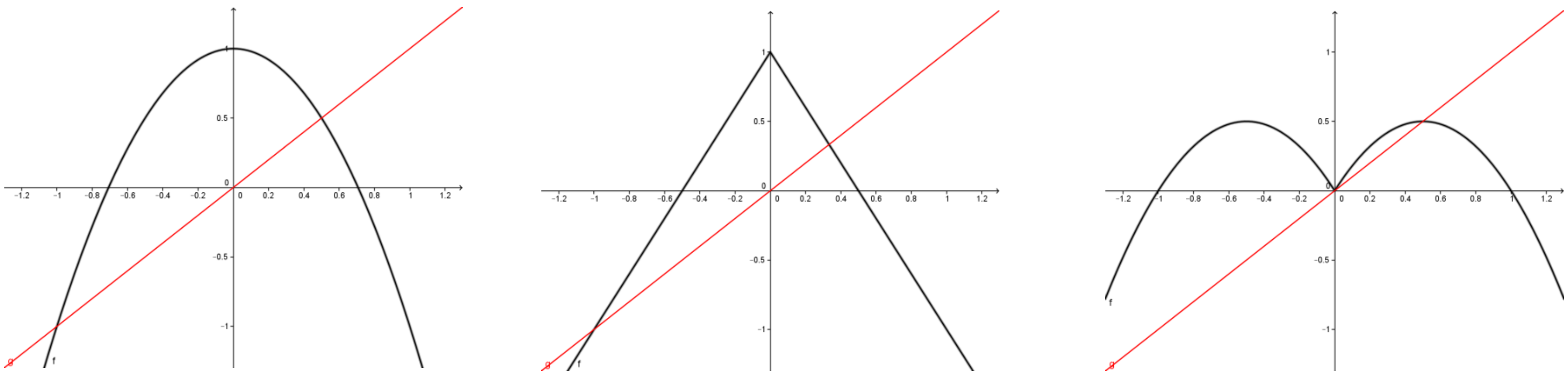
Another method: Tent-Logistic map

We introduced a combined Tent-Logistic map: TL_μ

$$f_\mu(x) \equiv TL_\mu(x) = L_\mu(x) - T_\mu(x) = \mu|x| - \mu x^2 = \mu(|x| - x^2)$$

When used in more than one dimension, TL_μ map can be considered as a two variable map:

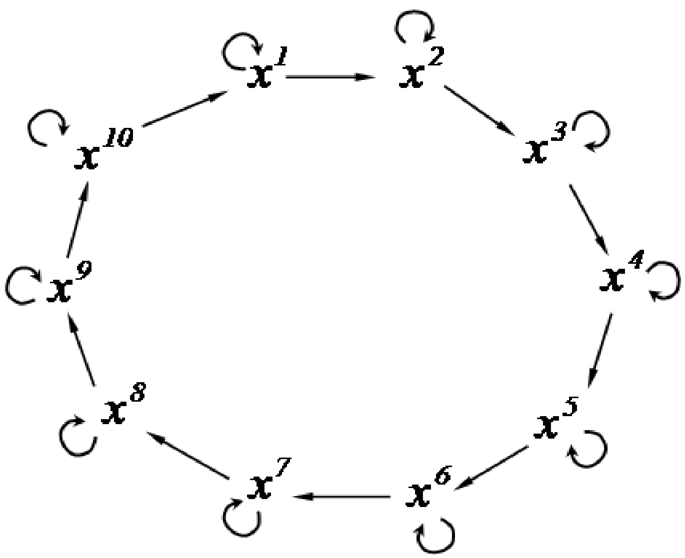
$$TL_\mu(x^{(1)}, x^{(2)}) = \mu(|x^{(1)}| - (x^{(2)})^2)$$



Ring coupling of several 1-D maps

Instead of using one single 1-D maps $f:[-1,1]\rightarrow[-1,1]$, it is possible to use simultaneously several (up to 10 or 20) 1-D maps coupled in a ring way.

Restraining the new p-dimensional map to the torus: $[-1,1]^p$



Ring coupling of Tent with Tent-Logistic maps

Hence it is possible to define a mapping: $M_p : J^p \to J^p$

where $J^p = [-1,1]^p \subset \mathbf{R}^p$

with the coefficients k^i set to -1 or +1

$$M_p \begin{pmatrix} x_n^{(1)} \\ x_n^{(2)} \\ \vdots \\ \vdots \\ x_n^{(p)} \end{pmatrix} = \begin{pmatrix} x_{n+1}^{(1)} \\ x_{n+1}^{(2)} \\ \vdots \\ \vdots \\ x_{n+1}^{(p)} \end{pmatrix} = \begin{pmatrix} T_\mu(x_n^{(1)}) + k^1 \times TL_\mu(x_n^{(1)}, x_n^{(2)}) \\ T_\mu(x_n^{(2)}) + k^2 \times TL_\mu(x_n^{(2)}, x_n^{(3)}) \\ \vdots \\ \vdots \\ T_\mu(x_n^{(p)}) + k^p \times TL_\mu(x_n^{(p)}, x_n^{(1)}) \end{pmatrix}$$

In order to maintain dynamics into the torus we use the injection:
$$\begin{cases} if\,(x_{n+1}^j < -1) \quad add \quad 2 \\ if\,(x_{n+1}^j > 1) \quad substract \quad 2 \end{cases}$$

Another 2-D chaotic PRNG

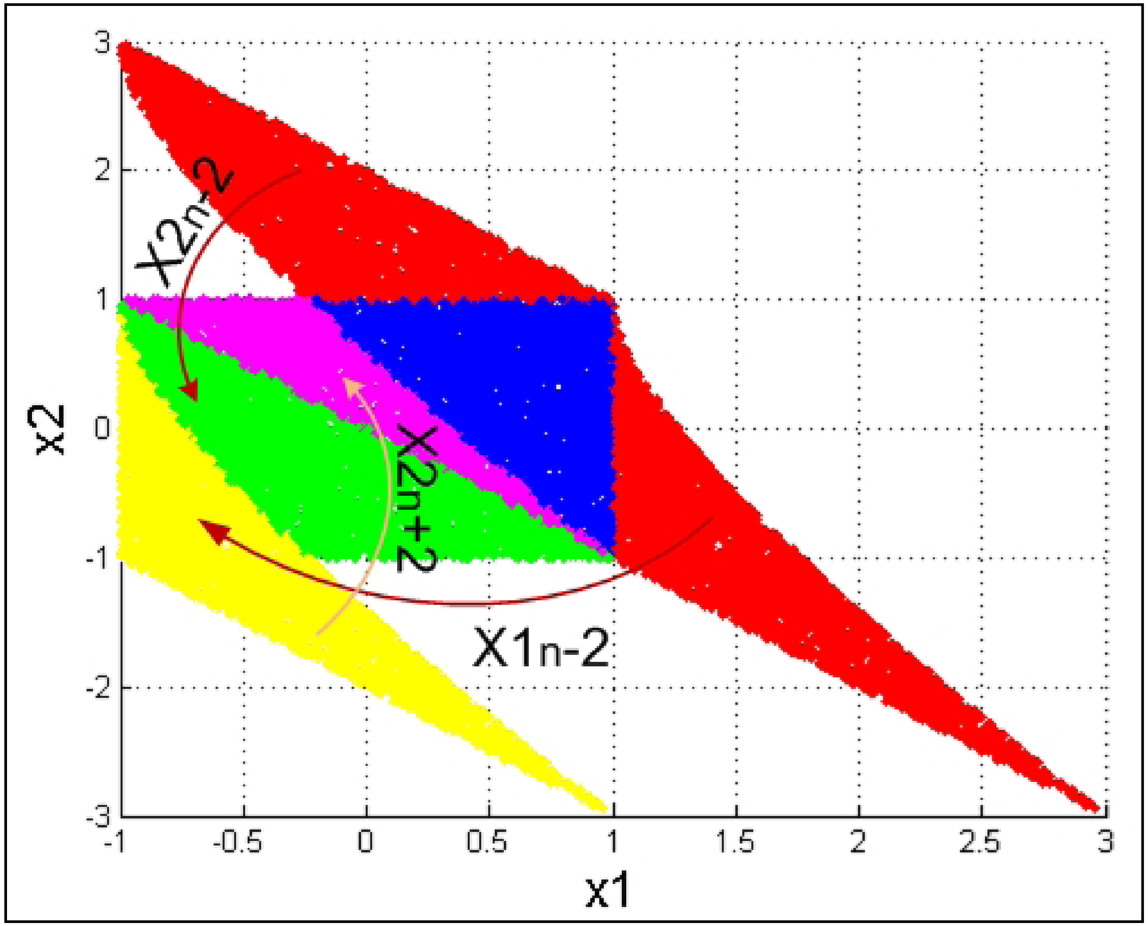
In order to improve the previous topologies, we define a new map with $\mu = 2$

$$MTTL_2^{SC}(x_n^{(1)}, x_n^{(2)}) = \begin{cases} x_{n+1}^{(1)} = 1 + 2(x_n^{(2)})^2) - 2\left|x_n^{(1)}\right| \\ x_{n+1}^{(2)} = 1 - 2(x_n^{(2)})^2 + 2(\left|x_n^{(1)}\right| - \left|x_n^{(2)}\right|) \end{cases}$$

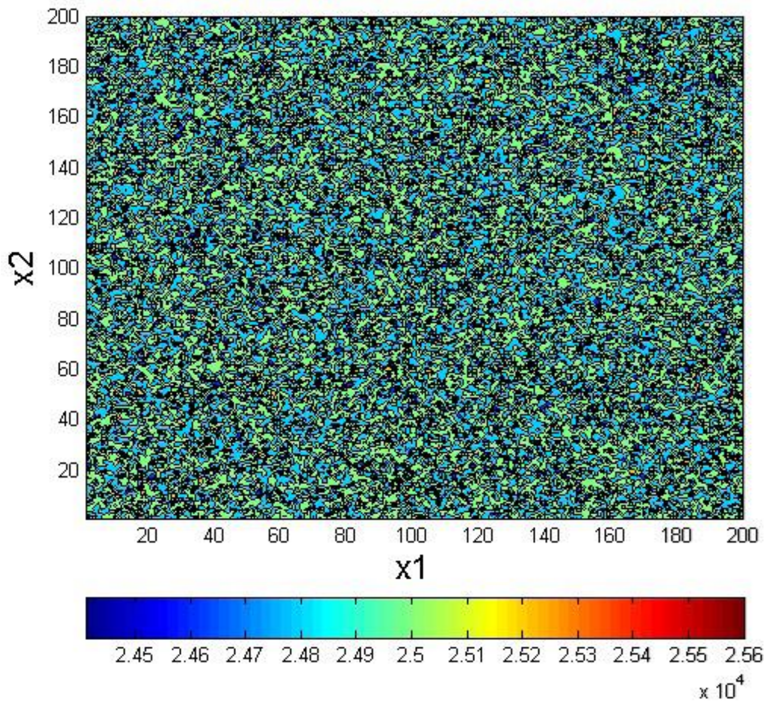
With a new injection mechanism which fits better the Torus

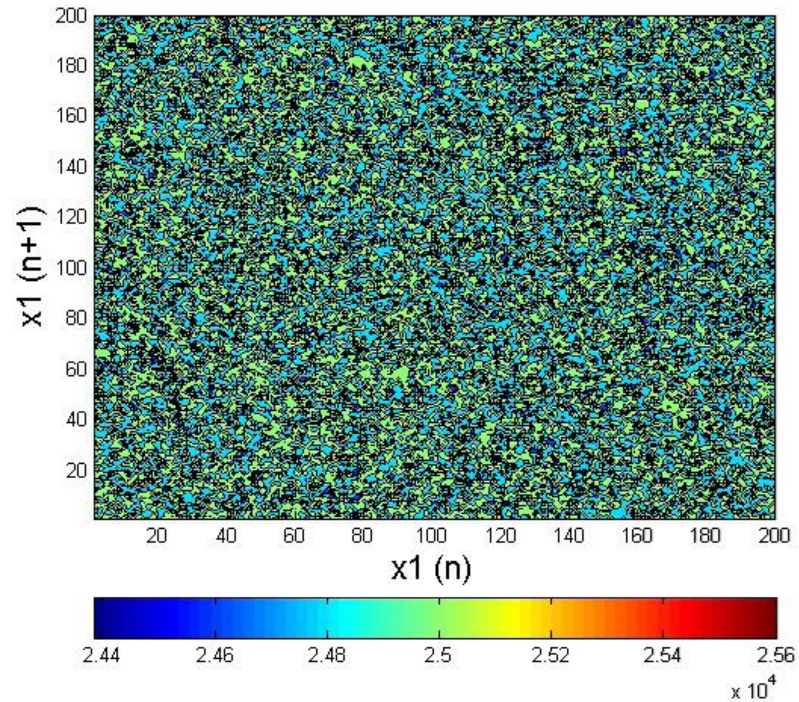
$[-1,1]^2 \subset \mathbf{R}^2$

$$\begin{cases} if(x_{n+1}^{(1)} > 1) \quad then \quad substract \quad 2 \\ if(x_{n+1}^{(2)} < -1) \quad then \quad add \quad 2 \\ if(x_{n+1}^{(2)} > 1) \quad then \quad substract \quad 2 \end{cases}$$



Injection mechanism of $MTTL_2^{SC}$ alternative map





Left: Approximate density function of $MTTL_2^{SC}$ alternative map, on the phase plane $(x^{(1)}, x^{(2)})$

Right: Approximate density function of $MTTL_2^{SC}$ alternative map, on the phase delay plane $(x_n^{(1)}, x_{n+1}^{(1)})$

Other numerical experiments using multi-core processor

Journal of Difference Equations and Applications

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How useful randomness for cryptography can emerge from multicore-implemented complex networks of chaotic maps

Oleg Garasym, Jean-Pierre Lozi & René Lozi

Published online: 15 Feb 2017.

These results show that the pace of computation is very high. When $TTL_2^{RC,5D}$ is the mapping tested, and the machine used is a laptop computer with a Core i7 4980HQ processor with 8 logical cores, computing 10^{11} iterates with five parallel streams of PRNs leads to around 2 billion PRNs being produced per second.

Since these PRNs are computed in the standard double precision format, it is possible to extract from each 50 random bits (the size of the mantissa being 52 bits for a double precision floating-point number in standard IEEE-754). Therefore, $TTL_2^{RC,5D}$ can produce 100 billion random bits per second, an incredible pace! With a machine with 4 Intel Xeon E7-4870 processors having a total of 80 logical cores, the computation is twice as fast, producing 200 billion random bits per second.

Chaotic Cryptography

Cryptography based chaos

Several hundred of publications in this domain, not only for cryptography but also for generating Hash functions (important for mining cryptocurrencies like Bitcoin, Ethereum…)

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Communications in Nonlinear Science and Numerical Simulation 10 (2005) 715–723

Communications in Nonlinear Science and Numerical Simulation

www.elsevier.com/locate/cnsns

Short communication

Cryptography using multiple one-dimensional chaotic maps

N.K. Pareek [a,b], Vinod Patidar [a], K.K. Sud [a,b,*]

ISAST TRANSACTIONS ON ELECTRONICS AND SIGNAL PROCESSING, VOL. 1, NO. 2, 2008 1

Multi-algorithmic Cryptography using Deterministic Chaos with Applications to Mobile Communications

Jonathan M Blackledge, Fellow, IET, Fellow, BCS, Fellow, IMA, Fellow, RSS

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Physics Letters A 366 (2007) 211–216

PHYSICS LETTERS A

www.elsevier.com/locate/pla

Theory and practice of chaotic cryptography

J.M. Amigó [a,*], L. Kocarev [b], J. Szczepanski [c]

Recent methods

These methods are not only mathematical but implemented of FPGA cards

applied sciences

MDPI

Article

Design, Implementation, and Analysis of a Block Cipher Based on a Secure Chaotic Generator

Fethi Dridi [1,2], Safwan El Assad [2,*], Wajih El Hadj Youssef [1], Mohsen Machhout [1] and René Lozi [3]

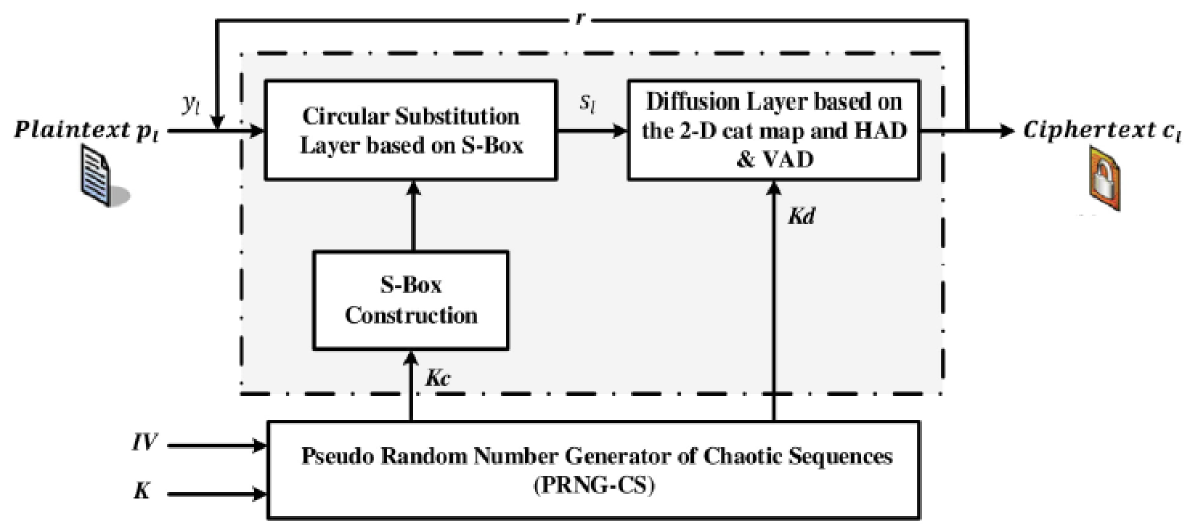


Figure 1. Diagram of the encryption process.

Chaotic Optimization

Random and chaotic optimization

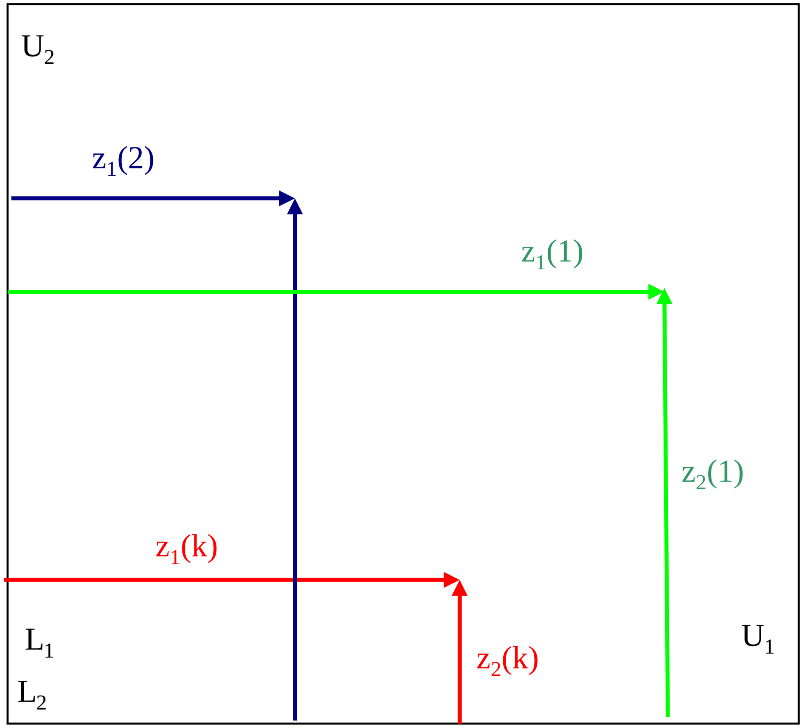
IEEE TRANSACTIONS ON EVOLUTIONARY COMPUTATION, VOL. 7, NO. 3, JUNE 2003 289

Chaotic Sequences to Improve the Performance of Evolutionary Algorithms

Riccardo Caponetto, *Member, IEEE*, Luigi Fortuna, *Fellow, IEEE*, Stefano Fazzino, and Maria Gabriella Xibilia, *Member, IEEE*

The space of variable is randomly explored by tossing random numbers for every variable.

In 2003, Riccardo Caponetto et al. introduced chaotic numbers in Evolutionnary Algorithms, they found more efficient than random numbers.



chaotic optimization in industry

In 2007, Leandro dos Santos Coelho used a Chaotic Optimization Method based On Lozi Map (COLM) he introduced few years before, for industrial application.

Available online at www.sciencedirect.com

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Chaos, Solitons and Fractals 39 (2009) 1504–1514

CHAOS SOLITONS & FRACTALS

www.elsevier.com/locate/chaos

Tuning of PID controller for an automatic regulator voltage system using chaotic optimization approach

Leandro dos Santos Coelho

Pontifical Catholic University of Paraná, PUCPR Industrial and Systems Engineering Graduate Program, LAS/PPGEPS Imaculada Conceição, 1155, Zip code 80215-901, Curitiba, Paraná, Brazil

Accepted 1 June 2007

PID controller

The Proportionnal-Integral-Derivative (PID) controller continues to be the main component in industrial control systems, included in the following forms: embedded controllers, programmable logic controllers, and distributed control systems.

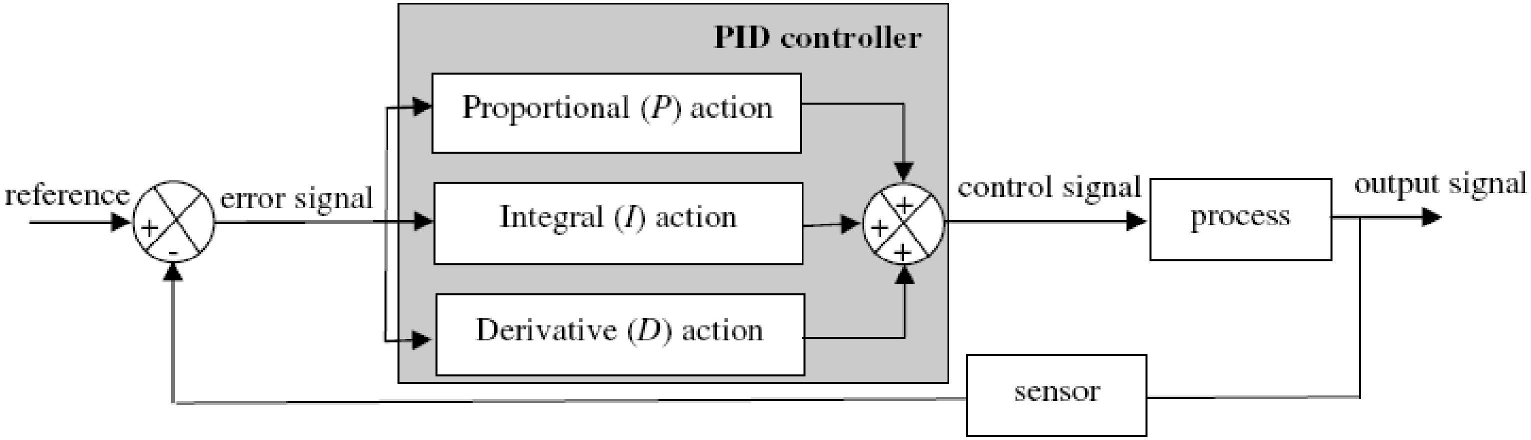


Fig. 1. Block diagram representation of a PID controller in a closed loop system.

It is reported that 80% of PID type controllers in the industry are poorly/less optimally tuned and that 30% of the PID loops operate in the manual mode and 25% of PID loops actually operate under default factory settings.

PID controller

As modelled in the paper of Coelho, the transfer function of PID controller (Fig. 1) is described by the following equation in the continuous s-domain (Laplace operator):

$$G_{\mathrm{PID}}(s) = P + I + D = \frac{U(s)}{E(s)} = K_{\mathrm{p}} + \frac{K_{\mathrm{i}}}{s} + K_{\mathrm{d}} \cdot s$$

where *U(s)* and *E(s)* are the control (controller output) and tracking error signals in s-domain, respectively; K_p is the proportional gain, K_i is the integration gain, and K_d is the derivative gain.

Tuning the PID is searching the values of K_p, K_i and K_d which minimize an objective function.

AVR (Automatic-Voltage-Reduction)

A simplified AVR system comprises four main components, namely amplifier, exciter, generator, and sensor . In the work of Coelho the AVR system is compensated with a PID controller. A block diagram of AVR system using PID control and chaotic optimization procedure is shown in Fig. 2.

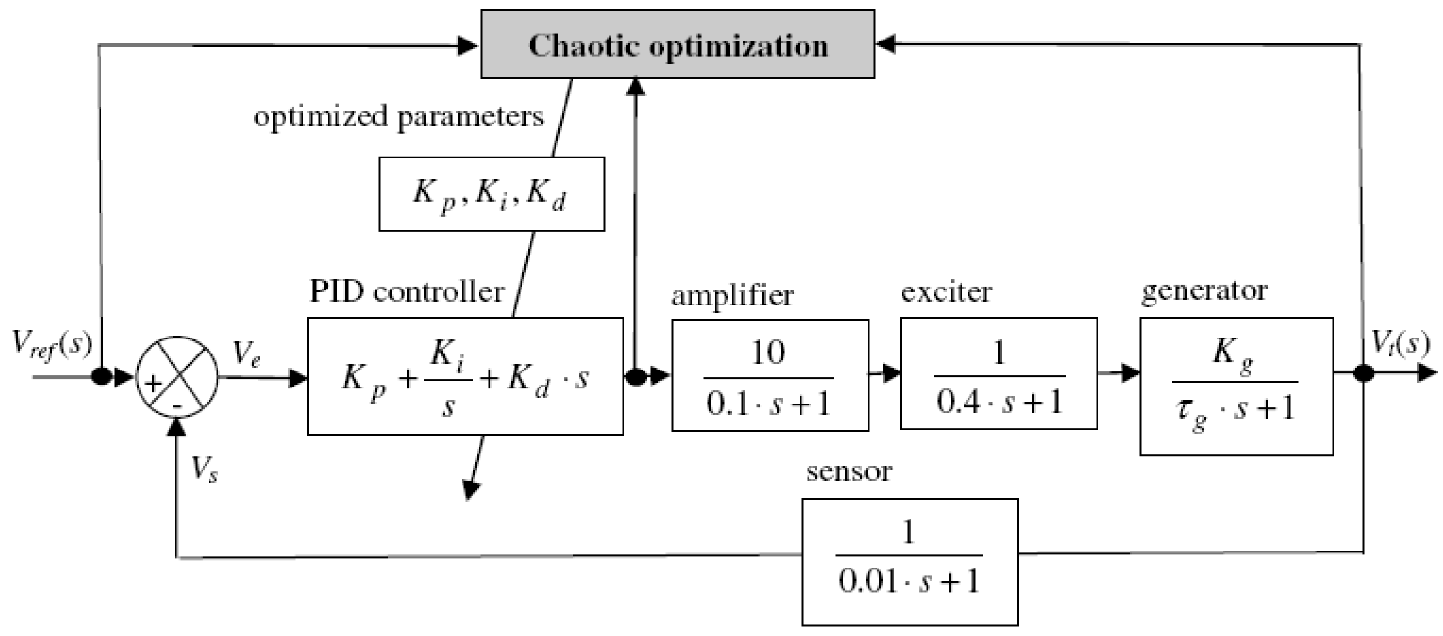
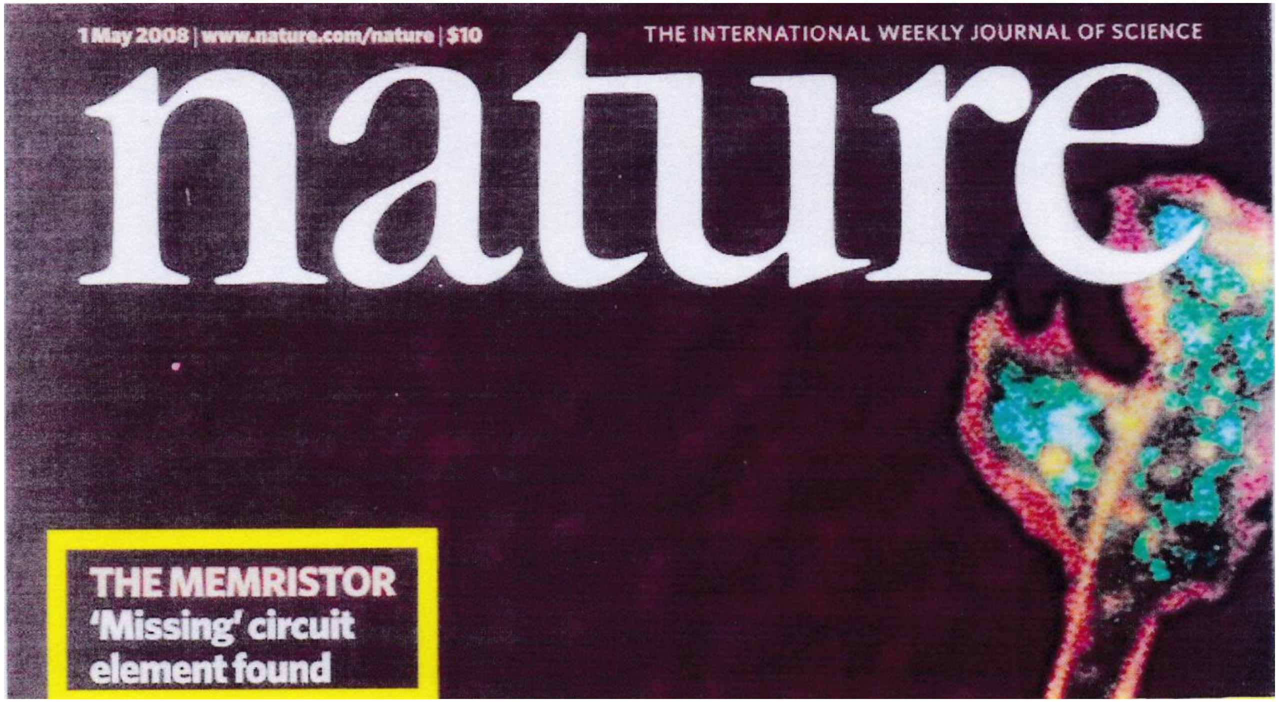
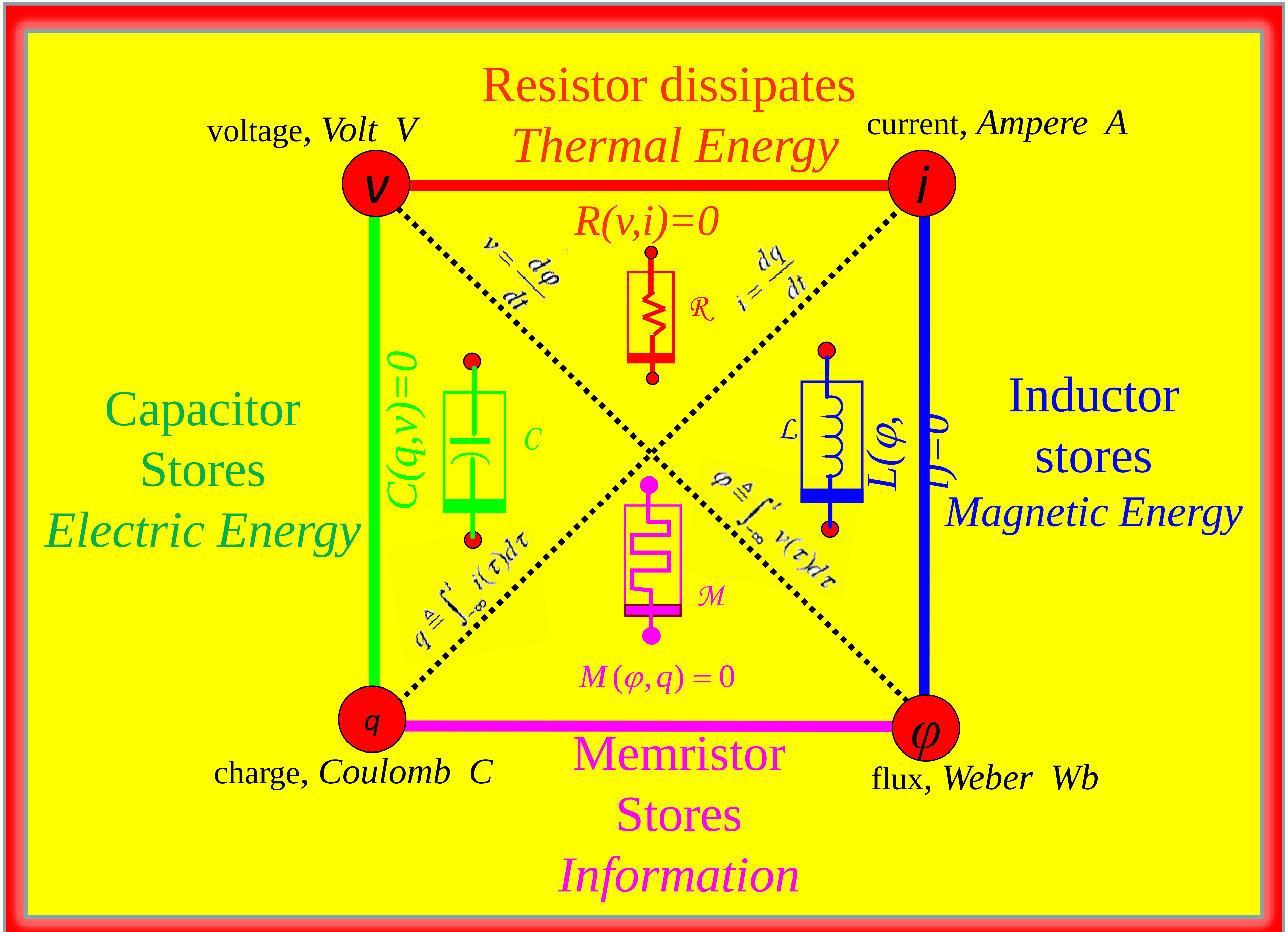


Fig. 2. Block diagram representation of an AVR system using a PID controller with chaotic tuning.

Memristors

Electric device invented by Leon Chua in1971, and realized in nanotechnology since 2008





Resistor dissipates
Thermal Energy
voltage, Volt V
current, Ampere A
v
i
R(v,i)=0
$v = \frac{d\varphi}{dt}$
$i = \frac{dq}{dt}$
ℛ
Capacitor
Stores
Electric Energy
C(q,v)=0
C
Inductor
stores
Magnetic Energy
L(φ,
i)=0
ℒ
ℳ
$\varphi \triangleq \int_{-\infty}^{t} v(\tau)d\tau$
$q \triangleq \int_{-\infty}^{t} i(\tau)d\tau$
$M(\varphi, q) = 0$
q
φ
charge, Coulomb C
flux, Weber Wb
Memristor
Stores
Information

Memristors

Among hundreds of memristor models some are linked to chaotic maps.

mathematics MDPI

Article

Memristor-Based Lozi Map with Hidden Hyperchaos

Jiang Wang, Yang Gu, Kang Rong, Quan Xu and Xi Zhang *

School of Microelectronics and Control Engineering, Changzhou University, Changzhou 213164, China
* Correspondence: zhangxi.98@163.com

2.3. Memristor-Based Lozi Map with no Fixed Points

To promote the chaos complexity of the Lozi map, a new 3-D memristor-based Lozi map is proposed by coupling the discrete-time memristor given in (3) into the original Lozi map described by (1). For the discrete memristor, the state variable y_n in the Lozi map is denoted as the input, and the state variable z_n is denoted as the internal state. Then the output of the discrete memristor becomes $v_n = y_n \sin z_n$, which is coupled to the second equation of the Lozi map after the gain k. Therefore, the memristor-based Lozi map can be constructed as

$$\begin{cases} x_{n+1} = 1 - a|x_n| + y_n, \\ y_{n+1} = bx_n + ky_n \sin z_n, \\ z_{n+1} = y_n + z_n, \end{cases} \tag{4}$$

where k is the coupling gain between the discrete-time memristor and the Lozi map.

Economy

Example of beautiful Figure from Commendatore, P., Kubin, I., and Sushko, I., 2015 , Typical bifurcation scenario in a three region symmetric new economic geography model. Mathematics and Computers in Simulation 108: 63–80

variables, $\lambda_{1,t}$ and $\lambda_{2,t}$. Taking into account the constraints, after dropping the time subscripts, the resulting dynamic system corresponds to a two-dimensional (2D) piecewise smooth map Z given by

$$Z : (\lambda_1, \lambda_2) \to (Z_1(\lambda_1, \lambda_2), Z_2(\lambda_1, \lambda_2)), \tag{9}$$

where

$$Z_r(\lambda_1, \lambda_2) = \begin{cases} 0 & \text{for} M_r \le 0, \\ M_r & \text{for} M_r > 0, \quad M_s > 0, \quad M_r + M_s < 1, \\ M_r \;\;/\;\; (M_r + M_s) & \text{for} M_r > 0, \quad M_s > 0, M_r + M_s \ge 1, \\ M_r \;\;/\;\; (1 - M_s) & \text{for} M_r > 0, \quad M_s \le 0, M_r + M_s < 1, \\ 1 & \text{for} M_r > 0, \quad M_s \le 0, \quad M_r + M_s \ge 1, \end{cases}$$

with $\begin{pmatrix} r = 1 \\ s = 2 \end{pmatrix}$ and $\begin{pmatrix} r = 2 \\ s = 1 \end{pmatrix}$,
$M_r \equiv M_r(\lambda_1, \lambda_2), \quad M_s \equiv M_s(\lambda_1, \lambda_2).$

Here the central Eq. (8) can be written as

$$M_1 = \lambda_1(1 + \gamma(K_1 - 1)), \quad M_2 = \lambda_2(1 + \gamma(K_2 - 1)), \tag{10}$$

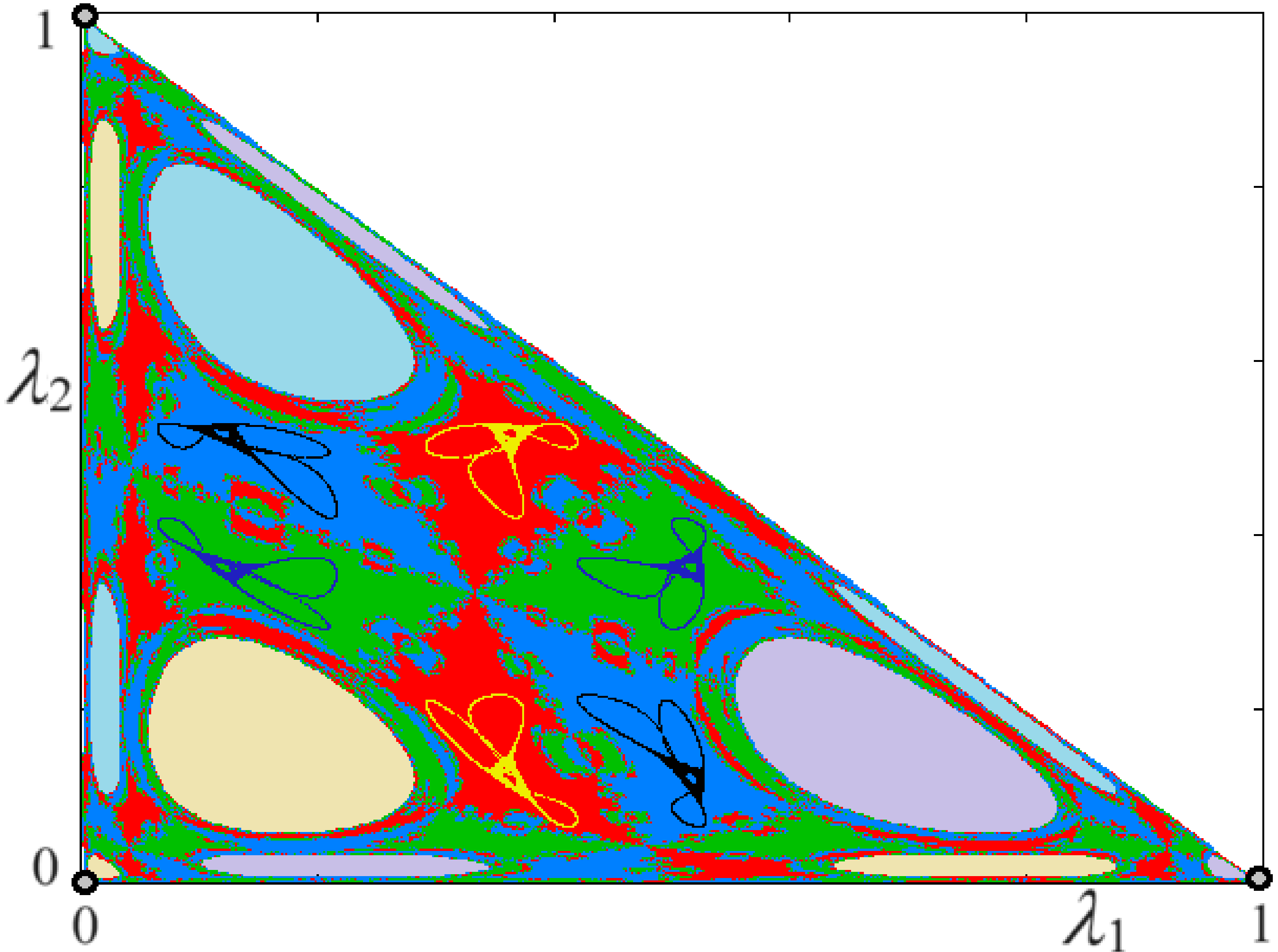
where

$$K_1 = \Delta_1^{\mu/(\sigma-1)} \frac{s_1/\Delta_1 + \phi(s_2/\Delta_2 + s_3/\Delta_3)}{D}, \quad K_2 = \Delta_2^{\mu/(\sigma-1)} \frac{s_2/\Delta_2 + \phi(s_1/\Delta_1 + s_3/\Delta_3)}{D},$$

The central equation of the dynamic system, holding for $r = 1, 2, 3$, is given by

$$M_{r,t} = \lambda_{r,t} \left(1 + \gamma \frac{\omega_r(\lambda_{1,t}, \lambda_{2,t}) - \sum_{s=1}^{3} \lambda_{s,t}\omega_s(\lambda_{1,t}, \lambda_{2,t})}{\sum_{s=1}^{3} \lambda_{s,t}\omega_s(\lambda_{1,t}, \lambda_{2,t})} \right),$$

Different basins of attraction



(a)

Thank you for your attention